III Doctrine of the conclusions

§ 58. Causal propositions

I take a class of composite truths and propositions that I have not mentioned until now as transition towards the doctrine of the conclusions. I mean the causal ones that we may express in the universal form: „Since A is, B is.“ We have repeatedly learned about cases where truths, be it actual ones or imagined ones, enter into the material of propositions, or rather, other truths. Thus is the case in common categorical propositions, in which a predicate of the subject that has beforehand been posited as existent is verified or denied; likewise in the universal propositions „All A are B“, the normal meaning of which is implied in that there is no one among the A that is not B. „Among the A“, that means, A exists. And this truth belongs to the material of the universal truth here as well.

Similarly it is the case with the causal propositions. The formula „Since A, thus B“ implies that A is, that B is; but not this alone, but simply that, since the A is true, the B is true as well. I do not need to say thereby of course that it is not the predications of the truth that need to form the antecedent and the consequent of the causal proposition.

The causal truth has some affinities with the hypothetical one, or rather, the causal proposition with the hypothetical proposition. Nay, most logicians seem to interpret the causal proposition as the respective hypothetical one, just combined with another proposition; namely, they seem to believe that the proposition „Since A is, B is“ is identical with the combination of both propositions „If A is, B is“ and „A is“. „Since God is just, thus the evils are punished“ = „God is just, and if God ...“ Yet we need to distinguish equivalence and identity. It is clear that the being of B is contained in the causal proposition exactly in the same way as the being of A. So that we needed to take three propositions then: „God is just“, „The evil ones are punished“, „And if ...“ But neither these three propositions complete the identity with the causal proposition. [233] Because it is clear indeed that the one considering as true these three propositions still does not need to judge that, since God is just ... What I mean probably becomes clearer if I express it in the following way: The one who judges causally wants to say that the truth, that the evil ones are punished, has its basis in the truth that God is just; or rather, the other way round, that the latter truth grounds the former one. Simply this thought was not expressed by the one merely saying that both propositions were truths, and if the one was accepted, the other one would be [accepted] as well. It would be conceivable that the judging one considers as true the combination of the three propositions here, and [that] he would not think of the causal relation here at all, or [that] he is not interested in it at all, [that] he thus does neither mean it. But certainly there is some equivalence, and not only between the three propositions and the causal one, but two propositions suffice, namely „A is accepted“, and „If A is accepted, B is accepted“. Every time we find two propositions of this form we may conclude that since there is A, there is B as well; and the other way round, if this is accepted, the other two propositions are accepted as well. And it is clear that by our ascertaining these connections we again express two causal truths.

This fact has its exact analogon in the universal propositions of the form „All A are B“ = „Among the A there is no <not-> B“, where we have an equivalent in the pair of propositions „A exists“ and „There is no A that is not B“, but still not a pair of propositions that is identical according to the thought. The inner relation, according to which the one truth is the foundation for another one, has been lost.

According to this analysis we stipulate the causal propositions as a proper class of propositions. We have quite a peculiar form of combining of actual or imagined truths at hand with them, and simply the one that constitutes the objective content of all conclusions.

<§> 59. Concept of conclusion

The logicians, when they start with the doctrine of the conclusions, explain, as we will not expect otherwise, the concept of the conclusion quite subjectivistically. They say: If we conclude that, since all men are mortal, and Cajus is a man, Cajus is mortal as well: What presents itself here? Well, the derivation of a judgment from certain given judgments, that is, a peculiar kind of generation of judgments, by the help of which the so called derived judgment comes about [234] by some preceding considering as true the so called premise propositions, [a consideration that is] still vivid in our consciousness. Or we say: With regard to the actual or supposed truth of the premise propositions the conviction of the truth of the *conclusio* comes about; a peculiarly complex state of consciousness is at hand, putting these judgments into one and characterizing the derived judgment in a peculiar way. Yet it is still unmistakable that the one saying that the truth B derives from the truths A does not want to state anything about the processes of his own consciousness, but about an objective relation between these truths, and this objective relation obviously is nothing else but the causal truth „Since A is accepted, B is accepted“. If the judging one has judged correctly, then this is an actual truth; if not, he presents it as truth, he pretends to express some truth, although this is not a truth. But, anyway, the opinion is not: „I have this and that process of consciousness within me“, but it is an objective truth that, since all men are mortal and Cajus is a man, he is mortal as well. The causal proposition does not speak of judging essences and their acts, but of a certain relation between the supposed truths „All men are mortal. Cajus is a man“ on the one hand and „Cajus is mortal“ on the other.

Like every conclusion, the word, objectively understood, is nothing else but a causal proposition, thus the other way round, every causal proposition can be named a conclusion in a certain way; but only if the term conclusion is taken in its widest sense. In general we use the term in a much narrower sense. In a certain way we may say [that] in the proposition „Since God is just, the evil ones are punished“ we derive the punishing of the evil ones from God‘s justice. And the one denying the proposition may also express himself in the form: I cannot see, not admit this conclusion. On the other hand you realize the difference between this example and the Cajus-example, where we talk of a conclusion in a narrower sense. Do also compare „Cajus is a man, therefore he is mortal“, and „All men are mortal, Cajus ...“. The first proposition is closely related to the complete conclusion, but it is itself not a complete conclusion, although a complete causal proposition. We will come back to this difference.

[235] Preliminarily we will allow us to apprehend the concept of the conclusion as widely as possible, in order to be compelled by the force of the matter itself to a restriction. We identify thus conclusion and causal proposition preliminarily.

Let us consider now, what may be meant by a theory of the conclusions, and what such a [theory] may be based upon according to its option. The totality of all conclusions is a self-contained manifoldness of truths, but an infinite one. It is impossible to name all conclusions, that is, all single causal truths, and to put them down in one logic. Neither would they belong into the logic. The single causal judgments belong into the single scientific fields. Obviously only the conformity to laws that reigns the causal judgments can belong into the logic, irrespective of the particularity of the field, for the case that there is something like conformity to laws. In what way are laws of conclusions to be understood now, and how are they possible? I already hinted at some points in the preface. If every conclusion was isolated in relation to any other, there would be <no> universal determinations of the conclusions, by which they would *a priori* be cognizable as accepted, then neither the talking of conclusion laws would make any sense. Determinations of the conclusions can only consist in the form of the propositions that emerge as premises or consequent in them. This same way of combining truths with truths is common to all conclusions. Therefore a separation into conclusion classes cannot be grounded on the causal form of combination, but only on the form of the combined truths. And if the form shall be determining for a conclusion law as well, then it needs to be accepted that every conclusion, combining the premises of a certain form F1, combined with a conclusion of a corresponding form F2, is a true one. Every conclusion law would need to be expressed in this way, only the concept F1 and F2 would change from law to law.

Of course we can take together arbitrary groups of propositions in the mere objectivation, and combine them in an arbitrary way for the formation of causal propositions, by taking any propositions as premises and any ones for conclusions.[[1]](#footnote-1)

<§ 59a.> Conclusion as law of hypothetical truths, not causal ones

We have made a consideration in the former lecture, in order to gain clarity on the question what could be meant by conclusion laws if we understand conclusions as causal truths. We said: If there are universal determinations of conclusions, by which they can be *a priori* cognized as actually accepted conclusions, then a universal truth must objectively exist, with the content that every conclusion, conspicuous by this quality, is true. If this truth shall have the character of a law then, then the questionable determinations of the conclusions can only emerge in the form or the content of the truths that are interconnected in the way of the conclusion, that is, causally as premise and conclusion truths. Since this causal form of connection is <common> to all conclusions as such, that which distinguishes them can only be implied in the connecting members, and thereby the universal as well, grounding a conclusion law. And if a conclusion law shall actually be grounded, only inner determinations of the premises,, and conclusion truths and [237] relations, exclusively grounding on these inner determinations, can be considered, that is, if we abstract from the universal moment of the truth itself, inner determinations, and inner relations of the corresponding propositions.

Every conclusion law thus would need to have the form: It is universally accepted that a causal proposition „Since A is, B is“ is a truth, if A has the inner determinations F1 in it, and B the corresponding inner determinations F2, or if there is a certain relation F(AB) between them that is grounded by these inner determinations.

We need to make some remarks on that. In any case, the following law is valid: Every causal proposition „Since A is, B is“ is universally a truth, in relation to which it is accepted that A is a truth, and at the same time the hypothetical proposition „If A is, B is“ is a truth; like the other way round a corresponding causal proposition belongs to every pair of truths of the latter form.

Subjectively spoken thus [this is what] takes place: If we know that A is valid, and that, if A [is] then B is, then we also know that we may conclude B from A. Obviously only those conclusion laws would have a value for us in practical relation, that do not have recourse to the truth of the B. Because we only want to conclude B from A with the help of the conclusion law. What matters for us is to find a way of justification leading us to B, after we already cognized A to be true. And thus that, which we strive for in practical interest is exclusively the finding of such laws, by the help of which the truth of those supplementing hypothetical propositions seems to be assured. What only matters for us therefore, for the purpose of cognition, are the laws of justification of the following form: It is universally accepted that a hypothetical proposition „If A is valid, B is valid“ is a truth, if A and B have these and those determinations, be it for them or in relation to one another. Such a law guarantees us the rightfulness of the proposition „If A, thus B“, relating to the *hic et nunc* given A and B in the given case. And if we know now already that A is valid, then we have reached everything, we also know that B is now accepted. But if we do not yet know the first, then it is our task to prove the validity of the A as well.

But does every prove need to be performed in this double step? Are there not any laws that make [us] cognize the validity of a causal proposition „Since A, thus B“ in one go, without our [238] need to take recourse to the truth of A, and to the truth of the corresponding hypothetical proposition? The answer is: Our practical procedure is completely justified. All laws, related to conclusions as such, are reduced to laws, related to the content of the hypothetical truths they imply. Probably the validity of the A may be one corresponding to the law as well; but, anyway, the law then exists, which presents the conclusion as one that is universally valid from two separate laws, of which the one says: Universally it holds true that every A of these and those determinations is true; and secondly: It is universally accepted that a hypothetical proposition „If A is, B is“ of these and those universal determinations is true.

Thus we are led back to the laws that govern the hypothetical truths.

If we possess all these laws, then we thereby have everything needed, in order to conclude new truths from the already given truths. And if we combine these laws with laws otherwise reigning the truth of any proposition classes, then we also have the causal laws, in which both is determined according to the law, the truth of the premises and the truth of the consequence. Since this is thus the case, the whole theory of the conclusions leads to a theory of the hypothetical truths; and indeed we understand by a conclusion theory in all logic, as a brief overview already shows, nothing else, although it has nowhere been stated clearly.

<§> 60. *Logical and alogical conclusion laws, and the corresponding classification of the hypothetical truths*

If such a theory of the hypothetical truths shall be possible now, if laws shall exist for these classes of truth, then obviously either all or certain groups of hypothetical truths need to be particularities of such laws. There need to thus exist cases, where a hypothetical truth is valid irrespective of the particularity of its material or certain components of its material, and thereby remains valid if these particularities, these components within it are varied in an arbitrary way. We cognize from examples that this actually happens in this way. The hypothetical proposition „If all men are mortal, and Socrates is a man, [239], then Socrates is mortal as well“ is a truth. But here is the peculiarity that we may arbitrarily vary certain moments of the material in this truth, namely the objectivations, or rather, concepts Socrates, man, mortal. We can for example take the centaur Chiron instead of Socrates. We can take the concept man, whatever we want, e.g. the concept horse, tree, and the like; and accordingly for the concept mortal. The single propositions, from which the material of the hypothetical truth constitutes itself by that may become wrong, ridiculous, absurd, the whole always remains an accepted hypothetical truth. A law is simply the basis here, which we express in the following form: „If all A are B and S is an A, then S as well is a B“; expressed equivalently: „If there is no A that is not B, and S is an A, then S is B.“ The letters designate variables here in an ulimited way, analogously like the letters in the arithmetical formulae. From this example we see that the objectivations of objects and concept objectivations that are variable here, constitute the logical unity of the proposition in such a way that no further objectivation enters its combination, and that the form of the combination is of a universally logical kind, i.e., has a certain meaning irrespective of the particularity of any classes of material. This is not always the case. If I compare three men, let us say Hans, Kunz, and Wilhelm, and conclude: Hans is taller than Kunz taller than Wilhelm, that is, Hans is taller than Wilhelm, then the conclusion is a particularity of a law as well. I can replace the three names by arbitrarily other ones, objectivations of arbitrary persons; arbitrary things may be named without deleting the truth of the proposition. But the relation of the height makes only „sense“ simply for that, which has a height. Spoken more concise: The relation of the heights, objectivated as a relation of non-heights, leads to an absurd objectivation. The relation of the heights thus depends on the particularity of the materials. The conclusion „a > b. b > c. Therefore a > c“ does not belong in the universally logical realm, it is limited to the field of the quantities.

Similarly, as objectivations of objects or concepts may be variable in an unlimited way in hypothetical propositions, it may also happen that whole propositions are like that. You see this immediately in the law, the truth of which is evident without further ado: „If A is valid, B is valid. If B, then C ...“ A, B, C represent any propositions here, and you realize at the same time that the variable propositions are only [240] combined in a universally logical way with one another. Nothing else enters the combination that restricts the use to special fields.

We see from these examples that there are manifold laws of hypothetical truths. Nay, we do not need to search artificially for the examples. Since it is valid as a fundamental proposition that every hypothetical truth, either in that way in which it is there, or according to a certain supplement, is the particularity of a universal law, so that each can be used for the constitution of a law. Division: I. Hypothetical truths, the particularities of laws, II. those that are no particularities of laws. This leads to a universal division of the hypothetical truths themselves: a) those that are based on universally logical concepts, b) those based on alogical concepts. Yet, endlessly many hypothetical truths will lead back here to one and the same law. Our examples lead us to an essential classification of the laws of hypothetical truths at the same time, or in short, of the conclusion laws. We will not be able to allocate all laws of that kind to the formal logic, since there are uncountable many ones among them that are based on the particularity of the field, in which they find the only possible use. Every hypothetical truth contains, as we know, variable terms, and each of the propositions it consists of, and into which such terms enter, may be regarded as a relation or combination of the respective variable objectivations. We may distinguish relations and combinations into two classes:

1. Into those that are of a universally logical kind, i.e. [those that] are possible for the objects of the highest logical categories, that is, for objects, concepts, propositions as such, and [that] are insofar based on these categories.
2. Into those that are based on any other concepts, e.g. on the concepts of the colour, the chime, the spatial extension, the time, and the like. The relation between object and concept, the predicative relation is a universally logical one, likewise the combination of single objects into sums, the combinations of multiple predicates into a conjunctive predicate, the combinations of multiple propositions into a single conjunctive or disjunctive or hypothetical proposition. In contrast to that [241] the relations of the prior and afterwards, of the right and left, the combinations of stretches into a figure, of fields into a bodily formation, and the like, are alogical.

Accordingly we distinguish the laws into logical and alogical ones. Only logical relations and combinations enter the logical laws, they constitute exclusively from the universal logical categories, and that which is based on them. Other concepts enter the alogical laws as well, and they need to be removed from them, for example by way of generalization. This holds also true for the conclusion laws that are likewise distinguished. The formulae, in which we express logical conclusion laws, apart from the letters as signs for unlimited variables, accordingly contain only syncategorematic signs that point to universally logical forms of combination and relation, probably to universally logical concepts themselves, although these need to be removed as well by way of an equivalent change. This is the case with the formula „If all A are B, then there are B that are A.“ We can also say for that: „If the totality of the A has the determination that every B is, and the like.“ These concepts totality, determination, etc., introduced by this equivalent circumscription, are only of a universally logical kind, they thus do not chnage the character of the law.

*<§> 61. The different classes of logical conclusion laws and theories*

The universally logical laws then again are distinguished into multiple groups: into laws that are based on the concept of the proposition, into laws that are based on the concept of the concept. These are then joined by further realms, intimately connected to the just mentioned ones, laws that are based on the concepts of the universally logical relations and combinations, therefore are based on the one hand in the concepts combination and relation as such, and again in the concepts multiplicity, number, order, and so forth. It may be surprising that the doctrine of the numbers that was mentioned here as well integrates into the universal logic, insofar as it contradicts the common opinion. But exceptional thinkers, I only mention Lotze here - and perhaps I had to add Leipniz here as well -, have cognized the fact rightly. Number is a specific differentiation [242] of multiplicity, and multiplicity represents the most universal logical concept of combination for objects as such.

Naturally those theories need to be put at the tip of the theoretical processing of the hypothetical truths, which systematically develop the conformities to the law that are based on the concept of the proposition. Each of the mentioned universal proposition groups has its theoretical unity. There are principles that are simply set, and derived laws that are deduced from the principles. But every deductive context is made of conclusions. And these conclusions are valid for the propositions that emerge in them, either because they are valid for propositions as such, or they are valid for them with regard to the constitution of the materials. Unnoticed those most universal laws that are valid for propositions as such play a part in almost all deductive contexts, and thus it is right to first of all justify them, in order to then be able to relate to them in the further theories. As the next step then follow the categories of the law based on the concept of the concept. Concepts emerge in all truths, and thus it is clear again that, apart from conclusions having their content in the particularity of the objects and concepts, those play their part, and an especially important part in a deductive context as well, that are valid for objects and concepts as such.

Thereby our way is predelineated.

<§> 62. *Preliminary remarks on the constitution of an apriori theory of the propositional conclusion laws*

We thus need to start with the propositional laws. But how shall we find these laws? Shall we regard all examples of conclusions that can be found in life and science, and see, whether they are of a universally logical kind, and if this, whether they are based on the propositions as such, that is, are valid independent of the particularity of the propositions? This of course would be a process that could not be performed. On top of that we can neither expect that all valid conclusions are immediately apparent to us as true, and that thus their conclusion laws are accessible for us on this way. And eventually there are endlessly many possible laws as we will convince ourselves, and we cannot deplete an infinite multiplicity [243] by way of an individual consideration. On the other hand it is certainly impossible to see how we, if not by that consideration of all individual conclusions that cannot be performed, should be able to proceed by way of trying. What does the arithmetician do, if he wants to justify his science? Does he have apriori thoughts that would secure him of the completeness of the arithmetic categories of laws that need to be derived? Absolutely not. How thus did an arithmetic science emerge? Well, by analyzing the first given arithmetic propositions the way man first found them. They found that certain relations are based on the concept of the number: Two numbers are respectively either the same or one is bigger or smaller than the other.

They furthermore realized that certain combinations are based on the concept of the number, at first the addition, based on that the multiplication and potentization, and again the inversions of these operations: the substraction, the division, the extraction of a root and logarithms. Certain simple and immediately apparent laws were then given along with the elementary combinations that were then led back through thorough analysis to a certain minimum number of laws that cannot be further reduced. And these basic laws then served as a foundation for the systematic deductions, on which ever new and new laws were based. How does the arithmetician know that the combinations he numbers there are all of the conceivable ones for numbers as such? The answer is: He does not know it with evidence at all, it may be supposed, since uncountable many mathematicians have thought about the matter. We may suppose that they would have found other ways of combination, at least if they are accessible to man as such. But there cannot be any certainty here. And however improbable it may be that no basic operations that are cognizable as such by man have escaped the scientific researchers, then the option still remains that there is something more behind the concept of the number than man can cognize. And likewise it is the case with the relations as well, and again likewise with the basic laws. The arithmeticians put an infinite effort into finding the minimum number of the arithmetic axioms. But whether one or the other is not merely resulting from the other ones without their noticing it, that is still [244] questionable. At least a systematic proof is missing until now. Is the arithmetics from all these reasons not a science? Or is it therefore a merely empirical science, having its roots in mere induction and empiricism? Neither that. Which process of trying ever brought subjectively closer the thought of the basic operations, basic relations, principles to the mathematicians: that, which they perform in arithmetics is not an empirical performance, not observation and attempt, not inductive universalization, but pure deduction. The operation and relation concepts, once grasped, are there, and as concepts they are something objective. The basic laws, once grasped, are there, and they are not empirical, but apriori basic laws. The psychological intermediations they brought closer to man are not conclusions, not inductions from experience.

That is, for that, which the mathematician offers, he may claim the name of science, and of apriori science. He may just not assert that that, which he offers as foundations, is from apriori reasons everything that exists with regard to the numbers in themselves and *a priori* as foundations. The gods may possibly be advanced, they possibly have a broader basis, but concerning that, which is accesible for us, which rests on the basic operations and the principles known to us, gods and men need to be in harmony, if it is correct at all that the arithmetics is a science.

Likewise it is the case with the propositional theory and with all we are going to explain. We cannot guarantee any completeness. The conclusions of the common life and of the sciences have quite early, in antiquity already, led to the formulation of conclusion laws, i.e., of laws for hypothetical truths. The comparing consideration of these laws and a looking around in the field of the sciences that have grown so much since then, is the foundation for the first ascertainments of the propositional theory. We analyze the conclusions, we show that the corresponding laws depend on one another in many ways, that we may deduce the one law from the other on the basis of an already known and simpler law of deduction; and thus proceding the number of the conclusion laws that are to be admitted as independent is limited. By our proceding thus, cognizing the more complex to be more complex, the simpler [245] to be simpler, we eventually arrive at a series of simple, or for our cognition simple, conclusion laws. We find here as well that certain relations and combinations are based on the concept of the proposition, and that certain elementary laws belong to each. And this systematic combination of these laws into purposes of further deduction, the introduction of forms of combination that are ever more combined into the simple terms of the basic laws, which, by way of their universality, may be determined arbitrarily, leads to new and newly deduced laws in an endless number. The difficulty here is only to demonstrate on every step that all the conclusions, mediating the deduction of laws from laws, are subject to the laws themselves that have been cognized as basic laws already, since, if this demonstration was missing, it would immediately come out that the number of the basic laws is not complete and to be amplified by at least a new one. And of course there is the task, as in arithmetics to create as systematically as possible the progress from basic laws to derived ones, and to create methods, according to which we indeed have the option, to solve every conceivable task in an ordered procedure, and to thereby deductively prove every conclusion that is presented, however complicated it may be, in practical purpose, i.e., to reduce it to the simple elementary conclusions, or to name for any system of premises that is presented, that, which is the purely logical consequence from that. The arithmetics as that science, in which the deductive contexts are analyzed most thoroughly can again give us a pattern for the more precise objectivation of the way new laws emerge from basic laws. The first law the arithmetician sets is: a+b = b+a. From this one alone, endlessly many deduced laws result simply by a and b being able to be replaced due to their universality, also through arbitrary combinations of universal terms. E.g., every summatory combination α+β, γ+δ+ε, and so on, always represents a number, whatever α, β... may be as numbers. We may thus take such a combination for a in our formula as well, that is, for example form (α+β)+(γ+δ+ε)=(γ+δ+ε)+(α+β), and in that all letters may again designate arbitrary numbers. We thus again have a formula, again a law. But we may substitute instead of these combinations arbitrary other ones that are universally accepted as representing numbers as such. And thus [246] one elementary formula already contains endlessly many. But there is not one arithmetic basic law, there is a complete series of them. The same holds true for each, and at the same time we see the option that, after we have derived a valid formula, new formulas again emerge from that by the use of other laws. Thus for example a second law of the arithmetics is: (a+b)+c=a+(b+c). But according to the first law, the law of change, it is accepted that (a+b)+c= c+(a+b). That is, since something equal replaced by something equal results in something equal: a+(b+c) = c+(a+b). If we formulate the proposition of equality as a special law, then the new formula has thus emerged by mere use of already fixed laws, through subsumption of the special case under this or that fixed universal law.

Ever new formulas also emerge in another way than by this subsumption of complicated forms of combination for the elementary laws, or in the way of equivalent transformation of combinations according to these laws. The issue of the inversion leads to other kinds of classes of tasks. An example will clarify what is meant. If we are given any form of composition, to which again a number corresponds, for example ax2 + bx + c, then a certain number belongs to it as a value for every system of numbers a, b, c, x. Supposing though that x is an unknown number, and only a, b, c are known, and supposing the whole composition has the value m, then the problem is: Is there always a numver x in such a way that ax2 + bx + c = m, and if this [is the case], how is this number connected to a, b, c, m? We call these riddles equations in arithemtics. And also the result of the equations, in the case there is one, only happens by recourse to the arithmetic basic laws. Stepwise no proposition is allowed for the result that is not subject to the basic laws. We neither need the least important conclusion here, which, if it is a special number conclusion at all, would contain something else than a subsumption under the basic laws.

We easily see now that this kind of progress and of justification is not only limited to arithmetics, but needs to be possible everywhere, where certain elementary relations and combinations are based in a field, and above that there are certain elementary laws governing these forms, thereby in the realm of the propositions as well, and of the concepts [247] as such. Because we know that there are forms of combination here that , in a universally accepted way, form new propositions from arbitrary propositions; just think of the conjunctive and disjunctive combination. We know that there are relations - I remind here of the hypothetical relation -, and again there are laws for these relations. Every conclusion law is itself such a law. Thus we overlook in advance that there need to be some elementary laws, which, themselves not derivable, are the basis of all derivations, and that endlessly many new laws need to result from the elementary laws through entanglement <?> of the forms of combination.

And if this is verified through the further execution, then we may expect that the practical habit of the arithmetic procedure, namely the calculation, needs to find its exact analogon in the formal discipline of the conclusions. Although we are not concerned with the practical here, it still appears good to me, to point out to it in advance, and also, to clarify the external habit of the procedure as well as protect [it] against misinterpretations. What characterizes the calculation in the realm of the number? Obviously the following that we, in order to solve a problem, in order to derive a proposition, do not need to think of the concepts themselves, but may content ourselves with combining sign and sign in a certain procedure that is limited by firm rules, replacing complexes of signs by other complexes of signs, etc. The calculation is an operating with the signs and not with the concepts themselves, and the result of the calculation is again something purely signitive at first, a certain combination of signs on the paper. But the interpretation of the final result simply gives the searched for proposition.

What is the reason now for this so to say blind mechanical procedure? How come that we may content ourselves with such ordered operations of signs, in order to solve any arithmetic problems? How come that we enlarge our cognition by moving signs on the paper, and discover new laws? The matter is implied in the *arithmetica universalis*, in the algebra, not quite different from the numeric arithmetics known to you. You know from you rown experience that additions, multiplications, etc., are performed in a purely mechanic way with decadic numbers, insofar as not even machines are used to deduce the results. And the machines to not think for themselves, no thought corresponds to the signs within the machine.

[248] It is not difficult to get clarity at least roughly, on the option of this computational procedures. Each number is named by a well distinguished sign in arithmetics. Likewise certain signs are introduced for the combinations: +, -, :, x, by which new numbers are definable from numbers. And again for relations between numbers: =, >, <. Every arithmetic proposition then names the content of a certain relation, whether between singly determined or between arbitrary signs, and by the help of the signs that are introduced a clearly outlined formation of signs corresponds to each proposition, e.g. a+b = b+a. Propositions with letter signs designate laws that are valid for arbitrary values of the letters. But if the concept of an arbitrary number a is replaced by that of an arbitrary combination of numbers, for example α+β or a/b, and the like, then the replacement of the letter a through the combination of letters α+β, etc. corresponds to it in the signature. Each basic law gives such a rule for the replacement of signs through signs at the same time according to its signitive side. I can apprehend a+b = b+a as a rule of signs: It is allowed to change the order in the addition combination of two arbitrary signs, or: It is allowed to replace an addition combination through another one with a changed order, but the same members. If any new proposition is derived with the help of the basic laws in pure deduction, that is, through mere subsumption under the basic laws, then a purely signitive process corresponds to that in an exactly parallel way, in that new and new sign complexions are gradually derived from a sign complexion throught the mere use of the sign rules that correspond to the basic laws. A result that must lead to some right proposition when interpreted conceptually then corresponds to each proper derivation. The reason for that is the exact parallelism between the intellectual operations and the sign operations. If we have fixed all basic laws so that there is no deduction of the field that would need anything else for its single steps than mere use of a basic law, mere submission under the universality of its rule, then there is neither a deductive derivation for which there is no computable analogon. And if we ascertain that only those combinations and replacements of signs are allowed that correspond to the basic laws, no other ones, then the pure play with the signs always only leads us to signitive propositions that are truths when contentually interpreted.

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<§> 63. *Some basic laws that need to precede all theories*

The universal consideration of our last lecture resulted in the necessity for a comprehensive development of the conclusion laws to proceed gradually. It seemed to us that as laws of the lowest step those need to be set first that are based on the concept of the proposition, in further consequence then the laws that rest on the concepts of the concept and of the object. Certainly, whether the corresponding theories are separated in a completely pure way, whether those laws do not engage with the theory of the propositional categories of the law that find their systematic place in the theory of the conceptual [category of laws], but play their part in the first one by their ascertaining rules for the derivation of the secondary one from the basic laws as well, we cannot know *a priori*. But anyway the one is clear that there needs to be a correct systematic order of the conclusion laws, in which the later doctrines are independent with relation to the earlier ones, in which thus no *circulus in demonstrando* is performed. But something more: We have to demand that the derivations themselves do not fall under the conclusion laws that shall only be proved by them immediately or mediately. But certainly a problem is named by that that had not been set, let alone solved until now. And it is a problem the successful solving of which is afflicted with quite extraordinary difficulties. Let us begin with the propositional theory, and if we arrive at the primitive basic laws through a systematic procedure of trial, then the ordered deduction leads to the different groups of derived laws. At first we would like to think that everything is in the order now, that the goals of such a theory have been reached. But no. Whereas we may confide ourselves to the natural inclination of our logical thinking everywhere else, without being forced to gradually account to ourselves for the forms of the conclusions that are being used gradually, it is quite different here. The basic laws we take as a starting point are conclusion laws. The derived laws again are conclusion laws. But the derivations themselves are conclusions as well, and as such [they] have their conclusion laws as well. What now, if we deduced a conclusion law in such a way that the law that is the basis for the conclusion law itself, was simply the [250] one that had to be derived? We deduce from law A law B. But the principle of the deduction to which we owe the result B simly presupposes the B or is itself of the form B. This does not imply a circle in the single case. There is a circular proof where we consider a proposition to be proved by demonstrating it to be a consequence of another one that we could only prove with the help of the proposition in question. Or there as well, where we prove the truth of a sentence B from that of a sentence A, while A itself already contains the B as an explicit or overlooked premise. In our case though the proposition that is to be proved is not a premise, but it only gives the principle, according to which the conclusion proving it procedes as well.

Still we could not allow for a systematic theory of the conclusions, in which conclusions were used at some place, the principles of which only emerge as doctrines at a later place. An ideal conclusion theory thus would have to offer the following image. Certain primitive axioms that cannot be derived one from the other serve as basic pillars. Then there are the doctrines, that is, derived conclusion laws. These derivations are themselves again conclusions or fabrics of conclusions. But if we dissolve any such fabric into the elementary conclusions, then we arrive only at those conclusions with the first doctrine that fall under the axiomatic principles as specializations. For the second doctrine the conclusion proving it can also have the form that has been proved as valid for the first doctrine, etc. In short, which proof ever we may test and analyze in theory, we will always find those in the series of axioms or of the laws that have previously been proved that justify it.

If we make clear to ourselves the essence of this fundamental task, and if we then go over to the attempt to gain its solution, then we will soon become aware of a series of propositions that need to first of all be set up, in order to be able to take just one step of the proof.

Supposing there were certain universal basic laws within a theory, no matter what, on which the deduction of further laws is based, then I ask: In which way do we need to understand this grounding? Obviously in the following way that we use the universally set up basic laws. For example, I conclude from the arithmetic law A + B = B + A that I may write (a - b) + c instead of c + (a - b). I need this replacement for a proof right now. I [251] may do this because the universality of the law of interchange implies that the universal A may designate any difference a - b in particular as well. And thus it comports itself as such. If we shall have use of the basic propositions for any proof then we simply have to use them. But what is meant by using? It means to conclude the truth of an implied case from the truth of the universal law. Thus it is a matter of fact that the conclusion principle, declaring as valid the transferance of the universal to the particular, among the very first basic propositions, and anyway, prior to the setting of any doctrine, needs to emerge.

But a second thing is implied in such a use and a proof as such. The first mentioned conclusion principle only runs: If A is a universal proposition and A‘ is a special case of A, then A‘ is accepted as well. The A though are basic laws in the de facto conclusion, that is, truths, and we do not conclude with the mere „if“, but with „because“: Because A is valid, and A‘ is a case of A, A‘ is valid as well. A second principle is obviously hidden in this: If A is valid, and if it is also accepted that „If A, then A‘“, then A‘ is valid as well. Or even more universal: If a proposition A is accepted, and if it is accepted above that that, if A is true, B is true as well, then B is accepted as well. This principle obviously is always used. If we derive new doctrines B from any basic propositions or doctrines A, then we have first set up the connection: If A is valid, B is valid. The further step is then: A are valid now, they are simply basic propositions or already proved propositions; thus B is valid as well. Only now is B a self-sufficient doctrine for itself. This principle is nothing else but that of the *modus ponens* of the traditional logic.

Still this is supplemented by some other laws that are needed in a theory right at the beginning. Thus for example, if we combine multiple basic propositions or doctrines, then we immediately need the proposition: If the universal A is valid, and if the universal B is valid, then the universal A and B is valid as well, i.e., the combination of both is valid, and vice versa.[[2]](#footnote-2)

And again the following is an imperative basic proposition: A and B shall be two universal propositions, they may <have> a universal relation to the objects u of any delimited class. Then we may say: Supposing it is valid that every u, for which the proposition A is true [252] makes also true the proposition B, then it is certain that, if A is true for every u at all, B needs to be true for every u as well. If it is for example valid for every square that, if it is factorable into two triangles, the sum of its angles needs to be four right [angles], then it is valid as well: If every square is factorable into two triangles then every square has four right [angles] for the angle sum.

<§> 64. *Designations for the purpose of the propositional theory*

These are thus the propositions that need to precede all other basic propositions, and therefore we also need them for the ascertainment of the propositional theory. In order to proceed now in the strictest way possible, and in order to be able to fix the propositions in the most conspicuous and clearest way possible, it will be necessary indeed to introduce signs that designate the relation and combination as well as the universal terms of the propositions according to the arithmetic or geometric [signs]. By our defining the signs unambiguously and using them strictly in the sense of the definitions, we escape the ambiguities of the verbal expression at the same time, and the errors springing from them. The matter is, in the theory of the propositional conclusions, to find the universal truths and to systematically develop those that are based on the concept of the proposition as such. Such universalities now will of course have a relation to the different forms of combination and relation that are conceivable for propositions as such.

Two propositions may respectively, whatever their other content may be, be combined conjunctively into a new proposition, for example „God is just, and the evil ones will be punished.“ We will designate this combination through writing the letters next to one another. AB thus means, if we take the majuscles as signs of any single propositions, the conjunctive combination of the proposition A and of the proposition B; to be read: A and B. Likewise we designate by A + B the disjunctive combination; the sign is thus to be read as: A or B. The + functions here therefore as a sign for the separation in a certain way, and not in that way we might have expected it, as a sign of conjunction. We need to indeed pay attention to the fact that we regard the disjunction as a primitive combination here, that is, as equal to: One of the two propositions is [253] true (example: „God is just, or the evils will be punished“). It is not determined whether just one of them is true, that is, if the one is true, the other one is false. In the common talking the small word „or“ is quite often, although not solely, understood exclusively. We thus take this harmful equivocation into our sign +.

A third elementary way to form one proposition out of two is the hypothetical one: If A, then B. But this way of combination represents the basic form of the relation between propositions at the same time. The acceptance of A implies that of B, and thereby both propositions are brought into some relation from which peculiar relative determinations spring for each: to be a basis and to be a result. We designate this fundamental relation in the following way:∈. We designate the important case, in which A ∈ B and at the same time B ∈ A as A = B.

Universally a negation and an affirmation correspond to each proposition A: It is true that A is; It is not true that A is. We may ignore the designation of the affirmation, since it is equivalent to the simple proposition. We designate the negation as the index o: Ao or else through overlining: A.

We still need signs for universal propositions. We are not used to especially designate the universality in arithmetics. We write a + b = b + a, and understand this is in the way that a formula shall be named by that: Universally it is valid ... Likewise we may also understand the written down algebraic relations formulalike. Yet it is still necessary in some cases to especially express the fact that a proposition is universally valid, especially if all terms are the variables. The universality is in each universal proposition related to certain variables, e.g. „It is universally valid that a man is mortal.“ The small word „a“ is a sign for the variable. I can insert Hans and Kunz for „a man“, and whomever, anything that is simlpy a man. Or, we have two variables in the arithmetic proposition „An even and an uneven number result in an uneven number in the sum“. „An even and an uneven one“ are the grammatical subject, what is meant is: no matter which. But only these single terms are the carriers of the universality in these propositions. If we hint at a proposition by the symbol f(xy ... ab ...), containing the terms x, y, a, b ... then we want to understand Πxy... f(xy ... ab) as the proposition: f... is valid for no matter how many xy ...

[254] By contrast we want to sign the particular proposition „There is x, y ... for which there is f“ by Σxy... f.

We designate the singular proposition „For the determinately given x‘, y‘ ... f is valid“ by contrast simply through f(x‘y‘...).

Still we need to express a difference: A proposition may emerge as a mere proposition or the respective truth may be meant by it at the same time. For example, if we say that together with the proposition „God is just“ the proposition „The evils are punished“ is given as well, that, if the one is valid, the other one is valid as well, then we have not declared these propositions to be valid themselves. If we simply say though: „God is just“, then we mean the truth „God is just.“ We will hint at the difference with the help of an exclamation mark. Therefore: A!

<§> 65. *The theory of the propositional laws*

Before I go on naming the fundamental laws that are the basis of all these conclusions, valid for the propositions as such, I want to recapitulate the laws that, as we have explained, need to precede all other ones.

We named the conclusion from the universal to the subordinate particular at the first place. I consider it desirable to give you an exact formulation of the law, in order to prevent misinterpretations. If we confer a proposition related to all quadrangles, to the squares „in particular“, then the particular is in another sense here something particular, than if we confer a proposition that is valid for all men to Socrates. The particular is something universal itself in the one case, in the second case it is not. We consider the principle only in that sense where the particular is something universal itself so that the concrete wording would be the following: If a proposition f is universally valid for arbitrary u, v ... z, and if it is at the same time valid <that> sequentially every u‘ is a u, every v‘ a v and finally every z‘ a z, then the proposition f is in particular valid for every u‘, v‘ ... z‘ as well.

The number of these variables may be arbitrary. Example: If a proposition is valid for every quadrangle as such, then it is also valid for each square, for each rectangle, for each trapezoid, and so forth, since every square is a quadrangle, and so forth. This principle enables us to confer each law that is valid for propositions as such to arbitrary combinations, disjunctions or hypo-[255]thetical combinations of propositions, that is, in quite a universal way, so that the resulting propositions again have the character of laws, of formulas. This was the law α)

We furthermore named the law β), according to which, if f is universally valid and g is universally valid, the unitary proposition f and g is universally valid as well, that is, in relation to all the variables at the same time that have priorily been assumed in f as well as in g, and the other way round. Therefore we write it as an equation. It is a combination of two selfsufficient laws. We will always use them separately. This law gives us the right to conjunctively combine two arbitrary propositional formulas into one formula.

The law γ) designates that, if the hypothetical proposition „If f then g“ is universally valid, we need to conclude that, if f is universally valid, g is universally valid as well. If we thus have a hypothetical formula, and if we know that the antecedent exists as a formula for itself, then we know that the consequent needs to have a formulalike validity.

I want to insert a supplement here: I want to add δ), a law that is that trivial that it is too easily overseen, namely: If A is true, and B is true, then the proposition A and B is true as well, and the other way round. A1B1 = (AB)1

Then we let follow:

1. A(A∈B) ∈ B (*modus ponens*)
2. (A∈B) (B∈C) ∈ (A∈C)
3. (M∈A) (M∈B) ∈ (M∈AB)
4. AB∈A

1st doctrine: (A = B) ∈ (A∈B)

2nd doctrine: (A = B) ∈ (B = A)

1. AB ∈ BA. Of course BA ∈ AB is valid as well. We only need to change the naming of the letters. We may now combine the two formulas into 1) AB = BA. But this is a doctrine 2).

If we consider closely what is included in the proof then it is the following:

Π (AB ∈ BA) Π (BA ∈ AB) ∈ Π (AB ∈ BA) (BA ∈ AB)

<∈ Π> (AB = BA)

Since each of the two universal propositions on the left is a truth, that is, both together as well according to δ), then we may conclude according to the *modus ponens* that the consequent is a truth as well.

<This results in> 3) (A = B) = (B = A)

[256] In order to derive further doctrines though, we need to add some important laws now that are typically used for the procedure of proving in almost all of the later cases.

The formula II reads: (A ∈ B)(B ∈ C) ∈ (A ∈ C)

Using the way of conclusion γ) (and this implies a step of the *modus ponens* and of the α at the same time) we have:

Π(A ∈ B)(B ∈ C) ∈ Π(A ∈ C)

According to β):

Π(A ∈ B) Π(B ∈ C) ∈ Π(A ∈ C) (doctrine 2)

This means: If we have two of these formulas of the hypothetical form that the final member of the first is of the same form as the first member of the second, then we may deduce a new right formula from that, which has the first member of the first formula as antecedent, and the last member of the second formula as the consequent.

We can make use of this important principle immediately, in order to prove a correlate to the law IV, namely doctrine 3: AB ∈ B.[[3]](#footnote-3) Of course this proposition is as obvious as our axiom IV. But true to our intent to only accept those axioms that cannot be deduced from those we have already assumed, we desginate our current proposition not as an axiom. It is easy to be proved:

1. AB ∈ BA; IV. BA ∈ B. Since according to the third doctrine formulas that are chaining result again in a formula through combination of the marginal members we have: AB ∈ B. *qu.e.d.*

4th doctrine: (ΠM ΠN) ΠP ∈ Π(MN)P

Proof:

(ΠM ΠN) ΠP ∈ ΠM ΠN (V)

ΠM ΠN ∈ ΠMN (β)

(ΠM ΠN) ΠP ∈ ΠMN (III)

(ΠM ΠN) ΠP ∈ ΠP (2)

According to III: (ΠMΠN) ΠP ∈ ΠMN ΠP ∈ Π(MN)P (γ). Thus, according to III the proposition itself.

5th doctrine: ΠA Π(A ∈ B) ∈ ΠB. Proof in the same way. I, II, III.

[257] Furthermore of particular importance:

6th <doctrine>: ΠM ΠN Π(MN ∈ P) ∈ ΠP. It designates: If I have two universally valid laws, M and N, and the universal result is valid that P would result from M and N then P as well is a universal formula.

The proof is easy. According to 4:

ΠM ΠN Π(MN ∈ P) ∈ Π(MN) (MN ∈ P). The righthand side is, according to 5: ∈ ΠP. That is, according to II, the proposition itself.

Similarly we also prove:

<doctrine> 6: Π(M ∈ A) Π(M ∈ B) ∈ Π(M ∈ AB) according to axiom III. But we do not need that any more since we have the 6th.

The meaning of these sentences with the sign Π is implied in that they allow for concluding with formulas in such a way as if they were singular propositions, as if the letters in them designated not variables but certain propositions. If we conclude in the same way as if the letters were determinate ones, and if a proposition is the result, then we may lay claim to it immediately as a universal formula. I called especially important doctrine 6‘s way of conclusion. Because based on <it> is, for example, the procedure of mathematics, according to which we may treat arbitrary formulas of numbers in the calculation in such a way, as if they were propositions with determinately given numbers. Each conclusion then results in a formula of numbers, and not, for example, in a particularly valid proposition of numbers.

We will immediately have to use it in our field in the

7th <doctrine>: A(BC) ∈ (AB)C, and vice versa. That is, equation.

Proof for the one half:

1. A(BC) ∈ A

3rd A(BC) ∈ BC ∈ B. IV. way of conclusion according to 6.

A(BC) ∈ B; A(BC) ∈ A ] III: A(BC) ∈ AB: Again according to 6.

Similarly we conclude: A(BC) ∈ C:

Because: A(BC) ∈ C; BC ∈ C; A(BC)∈C

That is, likewise (III): A(BC) ∈ (AB)C

Now the propositions 8, 9, 10, 11:

8th <doctrine>: (A=B) ∈ (A ∈ B); (A=B) ∈ (B ∈ A)

9th <doctrine>: (A = B) ∈ or = (B = A)

10th <doctrine>: (A ∈ B)(A = A‘) ∈ (A‘ ∈ B)

11th <doctrine>: (A ∈ B)(B = B‘) ∈ (A ∈ B‘)

Gradually the doctrines will be proved as such: Something alike substituted by something alike results in something alike.[[4]](#footnote-4)

VI.

12th <doctrine>: ...

13th <doctrine>: ...

14th <doctrine>: ...

Proposition 14 is followed by:

15th <doctrine>: ...

16th <doctrine>: ...

17th <doctrine>: ...

18th <doctrine>: ..., and the other way round, that is, in the same way.

Proof:

The principle VI has the same form AB ∈ C: If we use VI for it then we get:

...

This proposition though has again the form AB ∈ C: It is a formlua in that. The „A“ is again a formula in it. If we thus use the proposition 12, then the proposition is the result without further ado. On the lefthand side there are actually two correct propositions now. Therefore the righthand side is a correct proposition as well (*modus ponens*).

We can simply say the following: The sense of 12 is: If we have a formula of the form AB ∈ C, and if A is a formula in that itself, then B ∈ C is also a formula.[[5]](#footnote-5) The derived proposition now is a formula of the form AB ∈ C: Therefore B ∈ C is a formula in it.

Many other propositions are the result in addition to this, which I leave aside.

Eventually there is only one axiom left:

[259] VII. ...

Obviously the reverse then is valid as well: ...

Because: ...

Change of the order: ...

According to 17: ... Therefore as well: ...

And this is then joined by many kinds of doctrines.

Apart from the relations of contingency only the combination of the conjunction had emerged in the former doctrines. We will now go over to the laws that are related to disjunction and negation. We need new basic laws for that:

VIII. ...

IX. ...

X. ...

XI. ...

We need for the extension of the formulas to operations with formulas here:

ε) ... Only one half is valid.

ι) ...

According to 18: ...; doctrine 2°.

...; doctrine 3°.

Both are conversely valid as well.

Immediate result from XI: ... (<doctrine> 1°. Proposition of the excluded third). We only replace B by Ao.

Likewise the above propositions and their conversions will be the result. Apart from that further doctrines will be the result.

<doctrine> 4°: ...

<doctrine> 5°: ...

<doctrine> 6°: ... The same proposition may be added disjunctively on both sides in an implication.

<doctrine> 7°: ...

Likewise in equations: ...

<doctrine> 8°: ...

<doctrine> 9°: ...; corresponds to the law III.

The so called law of distribution is of particular importance: ...

Furthermore the proposition: ...

We prove one half in the following way:

According to 18: ..., multiply on both sides ...

[260]

Let us regard the righthand side:

According to VIII: ...; according to IX though ...; therefore according to 12: ...

If we combine this according to II with the proposition that has first been inserted, thus: ...

Proof of the conversion (much more laborious):

... If I substitute XIβ for A: Ao. But according to X: ... Therefore ... That is (3°), the negations on both sides alike as well. And obviously we can substitute euqivalence. Thus ...; ... But M + aao = M; therefore ...

I would like to add some remarks here.

We have set our formulas as completely universal. But they seem to be subject to a certain limitation, namely that that none of the expressions becomes senseless through any particularity. But this seems to be the case if we particularize the A and B in such a way that the antecedent and the consequent of a contingency becomes identical. „If A is valid, A is valid“, this does not, considered carefully, make any sense. We do indeed use such expressions quite often, e.g. „If I have commanded something, then I have commanded it“, „If I have said something, then I have said it“, and the like. But such expressions do not simply say that the antecedent and the consequent have the objective relation of the contingency, but we mean something different, e.g., my command is, once given, binding, I do not revoke the command, I remain consequently at that. Likewise „If I have said something, then I have said it“, i.e.: If I have said something, then I remain true to myself, I do not wish to and I will not revoke it. There are thus not only identities at hand, as the expression wishes to suppose. I doubt, whether we can make original sense to the proposition „If A is valid, A is valid“ while desisting from such secondary thoughts that do not belong here. Similar restrictions are implied in that the conjunctions and disjunctions of identical terms do not make any original sense. A+A: One of both is valid, A or A. AA: Both is valid, A and A. If we say: „2x2 is 4, and 2x2 is 4“, then we repeated the identically same proposition, but we have not performed any conjunction of objective propositions.

We can only free ourselves from these limitations by introducing certain conventions:

[261] We know that universally is valid: ... If we identify A and B, then the righthand side becomes = ... It thus makes good sense, the content of the proposition of the contradiction. We may therefore conventionally posit: ... This means that we lend the following meaning to the proposition of identiy „If A is valid, A is valid“: It is wrong that A is valid, and A is not valid.

Likewise we conventionally posit that ... By the help of this convention we may then easily prove that we can also draw an equal sign for the signs of identity and that then all formulas keep their validity in a completely unrestricted way. We may therefore operate computationally, irrespective of the members being identical or not within the combinations. These conventions here have a similar function as certain analoga in arithmetics, with the help of which the 1 and 0, that are no numbers in the original sense, are adjoint to the numbers with the help of certain conventions. Thereby we save much writing. We can then also understand the arithmetic formulas that universally that the letters can represent 0 and 1 as well as proper numbers.

Following these remarks the much talked about proposition of the identity, if we understand the mentioned hypothetical proposition of the identical antecedent and consequent by that, only another way of expression for the proposition of the contradiction; another way of talking or writing, being advantageous for certain purposes of the concluding, since it verifies a case of exception in a harmless way.

One more. Among our formulas we find one that does not have the form of an equality or relation of contingency. And likewise there are premises in the conclusions that are simply validities or invalidities, and those that have the form of contingencies. But we can also bring the simple validities into the form of contingencies.

Because it is part of the easily provable propositions that AAo = BBo = CC0 = ... Let us designate this product of combination that remains the same everywhere as 0. We thus have AA0=0. If ..., or ... now then M is wrong according to the *modus tollens*. And we can prove at the same time that vice versa as well, if M is wrong, ... needs to be, so that this proposition may occur as an equivalent for „M is not valid“. Similarly we can equivalently substitute the proposition „M is valid“ by [262] ..., in which I is defined as the common value of the chain A+A0 = B+B0 = ... A+A0 I1.

<§66.> *<The> theory of the conceptual conclusions*

These are the laws that regard the relations of any concepts with regard to their objects and the relations of objects to concepts in most universal universality, that is, laws, that are based in the concepts object and concept that are valid for object and concept as such.

What kind of relations come into question here? At first the relation between object and concept. If we designate by Γ a certain, but not more closely designated object, a certain something, and by small Latin letters any predicates, then we want to sign the proposition „Γ is a, has the determination a“ by: Γεa.

We can read this also in the form „a certain something, a certain object is a“, we can then also understand by the Γ „this house“, „Bismarck“, and the like, in the given case.

It is somewhat different, if we say simply and purely: „Something is a“ in that sense, that is to be expressed equivalently by „There is something that is a“, „There is an a“. Here the indeterminacy of the something belongs to the intention of the expression. The proposition „Plato is a gentle thinker“ is a particularization of the proposition „Something is a gentle thinker“. But with the latter proposition I simply do not mean the former one therefore. If we say: „Something is a gentle thinker“, we do not want, or not in the sense in question here, to talk of a certain man, of a determinate one, whom we simply do not designate concisely. (We have the same difference with predicates. If we say that an object has a determination, then it may be meant that it has a certain determination that is just not designated more closely, e.g. to be red. But it may also be meant that it has a determination simply and purely, i.e. there is a determination, that belongs to it; and this has quite another sense.) If we thus use the sign Γ, then this is a universal sign for any determinate object: Socrates, Plato, a certain triangle, and the like. But if we want to express the existential proposition „Something is a“, then we write Εεa, and for the existential proposition itself we write Σa, „There is an a.“ If we now consider the further basic relation that is hinted at in the expression: „If Something is a, then it is b“; an expression that has often been set as an interpretation of the formula „All a are b“ or „Every a is b“. If we say: „If something is a, then it is b“, then we of course do not mean a certain single object, whose being-a is the precondition for its being-b.[[6]](#footnote-6) Consequently we may also express the content of the proposition in the following way: ... By the sign Π we hinted at the fact that Γ is the carrier of universality.

The propositional principles α), β), γ) are particularizations of certain conceptual ones.

1. The principle of substitution: A law that is valid for arbitrary propositions, is also valid for arbitrary forms of propositions that are substituted to the single propositions, and thereby universally for the terms of these forms. This is a particularization of the universal principle of substitution: If a proposition is universally valid for arbitrary value systems Σα, then it is also valid for arbitrary special systems: ... The corresponding proposition for singular systems becomes valid in the use of the propositional theory for given propositions, but not in theory. But we need to consider closely. It might be in operating with formulas that the conclusion from the universal to the single would be used.

The conceptual proposition runs: ... (*modus barbara*)

1. ... = ...

Conceptually: ...=...

1. ...

...

If it is valid for every α that if it is A it is B, then the following is valid: If every α is A, then it is B. Proof: If it is valid for every α that if it is A it is B, then it is valid for every α that is A that it is B. (Likewise the other way round.)

[264] ... Or more concisely: ...

...; according to the principle of substitution. The proposition is already proved by that.

A system of values, a system of propositions fulfills the law G of the form ... = A system of values has the quality that if A is valid for it, B is valid for it as well = For a system of values, for which A is valid, B is valid as well.

If we have supposed the particular cases as principles then of course the conceptual formulas are the result together with the propositional theory as well (after supposing certain basic formulas), and thereby the universal principles, the *modus barbara*, etc. The proposition ... as well;

... = ... = ...

If the particularizations are not put at the top of the propositional theory as principles then we would have to set the mentioned conceptual ones as principles. Therefore

1. *modus barbara*
2. ...
3. ... = ...
4. That is ... = ...

or just = ...

Here the proposition is used: Something alike substituted for something alike results in something alike.[[7]](#footnote-7)

1. *Deleted* But in general we thereby do not get any causal truths. These will neither be causal truths if we take a realm of given truths and form causal propositions from them in an arbitrary way. The question will thus be: Are there any universal form properties of causal truths, by which they will immediately be cognized as truths? Is every causal truth of some given material a particularity of a general law, which says: Such premises universally ground such a conclusion; to which we immediately add: Thus it is also accepted for this case, for this quite special material?

   But we could make the matter a bit easier. A hypothetical truth corresponds to every causal one as we know. And obviously the conformities to laws of the one and the other classes of truths need to correspond to each other exactly. Every causal proposition is equivalent to a hypothetical one in conjunction with another proposition, which takes over the truth of the hypothesis. If thus the hypothetical proposition is given to us, we know that, if its hypothesis is true, the conclusion from the hypothesis to the thesis would be accepted as well. And something analogous is accepted of the laws.

   We can thus restrict ourselves to the determination of the laws at first, which relate to hypothetical connections between propositions. And if there are universal laws of this kind at all, then classes of cases need to be provable, where a hypothetical truth is accepted irrespective of the particularization of its material, in other words: where any objectivations, possibly laws, may be varied arbitrarily in a hypothetical truth, without the truth stopping to exist. We see from some examples that this is actually the case. The hypothetical proposition „If all men...“ corresponds to the conclusion „Since all men are mortal ...“

   We cognize immediately that the material contains multiple unlimited variables. We may posit whatever for the objectivations man, mortal, Cajus, always a hypothetical truth will be the result. The universal law is simply accepted: „If all A are B, and S is A, then S is B as well“, no matter what S, A, B may mean. [↑](#footnote-ref-1)
2. *Deleted* And for the disjunctive combination the same holds obviously true. [↑](#footnote-ref-2)
3. 4th: (A=B) ∈ (B∈A) (according to 3). [↑](#footnote-ref-3)
4. (12th <doctrine>: (A ∈ B) ∈ (AC ∈ BC)

   AC ∈ A; A ∈ B; AC ∈ B. Now according to <doctrine> 3): AC ∈ C; AC ∈ BC

   (AC ∈ A)(A ∈ B) ∈ (AC ∈ B); (AC ∈ A) ∈ ((A ∈ B) ∈ AC ∈ B)

   Π formula Π: (A ∈ B) ∈ (AC ∈ B); (A ∈ B) ∈ AC ∈ C (formula); (A ∈ B) ∈ (III)(AC ∈ B) (AC ∈ C) ∈ AC ∈ BC. *quod erat demonstrandum*.)

   *Behind that deleted* Prior to that new axiom:

   1. (AB ∈ C)A ∈ (B ∈ C)

   This results in different important consequences: View below 12.

   E.g. 13th AB ∈ C ∈ (A ∈ (B ∈ C): Once ((AB ∈ C)A ∈ (B ∈ C))(AB ∈ C) ∈ (A ∈ (B ∈ C)).

   The first part [is] a formula, the whole as well; that is, in the sense of the way of conclusion VI of the propositions. (We can repeat the formation once more, and we then have two formulas as premises.)

   14th A ∈ (B ∈ A). Then we replace C by A in VI, then (AB ∈ A) (formula) A ∈ (B ∈ A)

   12th We can set the proposition as such ΠAB ∈ C ΠA ∈ Π(B ∈ C); according to β and γ. [↑](#footnote-ref-4)
5. The proof of the inversion according to <doctrine> 14. On the righthand side both sides are multiplied by B. [↑](#footnote-ref-5)
6. *Deleted* Nor do we mean that the result of the proposition „Something is a“ („There is an a“) was, „The same Something is b“. Because, if in the antecedent the Something is taken in the undetermined sense, then there is no identity that can relate to that. It is not meant that, if a Something that is not designated more closely is a, the same a is b as well, but obviously it is meant that it is valid for each Something that is a, that it is b as well. It is meant that the Something is an independent variable in the antecedent, and that the Soemthing in the consequent has to to run through identically the same values. [↑](#footnote-ref-6)
7. How about that? Is that a result of the propositional theory or an axiom? [↑](#footnote-ref-7)